Boundary-Layer Flow Past a Cylinder with Massive Blowing

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Introduction

CORRELATIONS of the unsteady laminar compressible stagnation point boundary layers with massive blowing rates are required in several applications. Of specific interest is the development of the heat shield for the Jovian probe.[1] For massive blowing rates, there is a considerable increase in the overall boundary-layer dimension in the thermal and velocity gradients found in the neighborhood of the surface. The effect of large injection rates on the steady laminar compressible boundary-layer flow on the stagnation line of an infinite swept cylinder has been studied by Libby and Kassoy.[2] However, the effect of massive blowing on the analogous unsteady case has not been reported in the literature.

The aim of this Note is to study the effect of massive blowing on the unsteady laminar compressible boundary-layer flow along the stagnation line of an infinite swept cylinder when the freestream velocity and blowing rate vary with time. The governing partial differential equations have been solved numerically using an implicit finite-difference scheme in combination with the quasilinearization technique. The results have been compared with the available results.

Governing Equations

Let us consider the unsteady laminar compressible flow on the stagnation line of an infinite swept cylinder. We assume that the external flow is homentropic and that the surface is maintained at a constant temperature. We further assume that the inviscid flow velocity components vary arbitrarily with time. Under the foregoing assumptions, following Beckwith,[3] the boundary-layer equations governing the flow can be expressed as

$$(NF'')' + \varphi f f'' + \varphi[(\rho_e/\rho) - f'^2] + \varphi^{-1}\varphi_{t^*}[(\rho_e/\rho) - f'] - f'_{t^*} = 0 \tag{1}$$

$$(NS')' + \varphi f S' + \varphi^{-1}\varphi_{t^*}[(\rho_e/\rho) - S] - S_{t^*} = 0 \tag{2}$$

$$Pr^{-1}(Ng')' + \varphi f g' + (1 - Pr^{-1})[(1-t_s)/(1-t_w)]\varphi^2 \times [N(S^2)']' - g_{t^*} = 0 \tag{3}$$

with the boundary conditions

$$f = f_w,\ f' = S = 0,\ g = g_w \text{ at } \eta = 0$$
$$f' = S = g = 1 \text{ as } \eta \to \infty, \qquad \text{for } t^* \geq 0 \tag{4}$$

The initial conditions are given by the steady-state equations by putting $\varphi = 1$ and $\varphi_{t^*} = f'_{t^*} = S_{t^*} = g_{t^*} = 0$ in Eqs. (1-3). We have used the following transformations:

$$\eta = \left(\frac{a}{\rho_e \mu_e}\right)^{1/2} \int_0^z \rho \mathrm{d}z, \quad u = ax\varphi(t^*) f'(\eta, t^*), \quad t^* = at$$

$$v = v_\infty \varphi(t^*) S(\eta, t^*), \quad w = -\left(\frac{\rho_e}{\mu_e a}\right)^{1/2} \frac{f\varphi + \eta_{t^*}}{\rho} \tag{5a}$$

$$G(\eta, t^*) = \frac{H - H_w}{H_e - H_w}, \quad t_w = \frac{H_w}{H_e}$$

$$\frac{\rho_e}{\rho} = \frac{(1-t_w)g + t_w - (1-t_s)\varphi^2 S^2}{\alpha} \tag{5b}$$

$$t_s = \lambda^{-1} = \frac{1 + [\gamma - 1)/2]M_\infty^2 \cos^2\Lambda}{1 + [\gamma - 1)/2]M_\infty^2}$$

$$N = \frac{\rho\mu}{\rho_e \mu_e} = \{\alpha^{-1}[(1-t_w)g + t_w - (1-t_s)\varphi^2 S^2]\}^{\omega - 1} \tag{5c}$$

$$A = -\left[\frac{(\rho w)_w}{(\rho_e \bar{u}_e)}\right] Re_x^{1/2}; \ \bar{u}_e = ax, \ a = \left(\frac{\mathrm{d}u_e}{\mathrm{d}x}\right)_{t^* = 0}$$

$$Re_x = \frac{\bar{u}_e x}{\nu_e}, \ f_w = \frac{A}{\varphi(t^*)}, \ \alpha = 1 - (1-t_s)\varphi^2 \tag{5d}$$

where x, y, and z are the principal, transverse, and normal directions, respectively; η the transformed normal coordinate; t and t^* the dimensional and dimensionless times, respectively; f' and S the dimensionless velocity components in the x and y directions, respectively; H and g the dimensional and dimensionless total enthalpies, respectively; φ an arbitrary function of time t^* representing the nature of the unsteadiness in the freestream; Λ the sweep angle; t_s $(t = \lambda^{-1})$ the sweep angle parameter; M_∞ the freestream Mach number; N the ratio of the density viscosity product across the boundary layer; t_w the dimensionless wall temperature; ρ, γ, μ, and ν the density, ratio of specific heats, viscosity, and kinematic viscosity, respectively; and Re_x the local Reynolds number. The subscripts e and w denote conditions at the edge of the boundary layer and at the wall, respectively; the prime derivatives with respect to η; and subscript t^* the derivatives with respect to t^*. The parameter ω is the index in the power law variation of the viscosity across the boundary layer; $\omega = 0.5$ for high-temperature flows and $\omega = 0.7$ for low-temperature flows. If the normal velocity at the wall $(w)_w$ is selected in such a manner that $-[(\rho w)_w/(\rho_e \bar{u}_e)]Re_x^{1/2}$ is a constant, say A, then the surface mass transfer f_w will vary according to Eq. (5d). $A < 0$ for injection. The Prandtl number Pr is taken as a constant across the bounday layer because, in most atmospheric flight, the change in Pr is small.

The skin-friction coefficients in the x and y directions $(C_f, \bar{C}_f)$ and heat-transfer coefficient St can be expressed in the form

$$C_f = \frac{2\tau_x}{\rho_e (u_e^2)_{t^*=0}} = 2Re_x^{-1/2} N_w \varphi f''_w$$

$$\bar{C}_f = \frac{2\tau_y}{\rho_e (u_e^2)_{t^*=0}} = 2Re_x^{-1/2} \left(\frac{v_e}{u_e}\right) N_w \varphi S'_w$$

$$St = q_w / [(H_e - H_w)\rho_e (u_e)_{t^*=0}] = Pr^{-1} Re_x^{-1/2} N_w g'_w$$

It may be noted that Eqs. (1-3) reduce to steady-flow equations by the insertion of $\varphi(t^*) = 1$ and $\varphi_{t^*} = f'_{t^*} = S_{t^*} = g_{t^*} = 0$, the results of which are essentially the same as those studied by Libby and Kassoy[2] and Beckwith[3] for large and small rates of injection, respectively.

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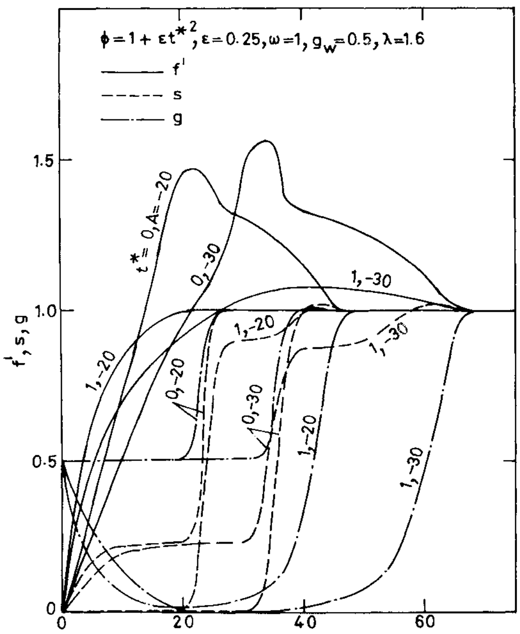

Fig. 1 Effect of blowing on the velocity and total enthalpy profiles.

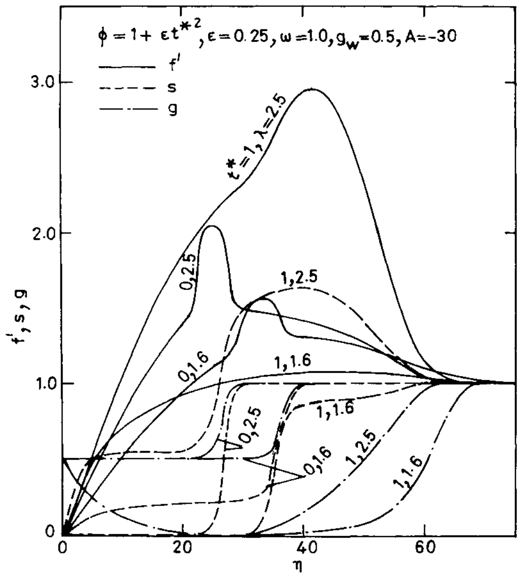

Fig. 2 Effect of the Mach number/yaw/angle parameter on the velocity and total enthalpy profiles.

Results and Discussion

Equations (1-3) with boundary conditions [Eq. (4)] and the initial conditions given by the steady-state equations have been solved numerically using an implicit finite-difference scheme with a quasilinearization technique with variable step size. The method is described in detail in Ref. 4.

Computations have been carried out for various values of the parameters and for two different unsteady freestream velocity distributions given by

$$\varphi(t^*) = 1 + \epsilon t^{*2}$$

and

$$\varphi(t^*) = [1 + \epsilon_1 \cos(\omega^* t^*)]/(1+\epsilon_1)$$

Such a distribution of freestream velocity may be relevant in space flight, flow over rotating helicopter blades, or about fluttering wing sections, etc. These velocity distributions represent accelerating/decelerating and fluctuating flows, respectively.

In order to assess the accuracy of the method, we have compared our skin-friction and heat-transfer results (f''_w, S'_w, g'_w) for the steady-state case with those of Refs. 2 and 3 and found them in excellent agreement. The comparison is not shown here for the sake of brevity.

The effect of blowing ($A<0$) on the velocity and total enthalpy profiles (f', S, g) for two values of time t^* is shown in Fig. 1. We observe significant overshoot in the x component of the velocity f' and the magnitude of the velocity overshoot increases as the blowing rate ($A<0$) increases. However, the velocity overshoot decreases as t^* increases because we have taken the accelerating freestream velocity distribution [$\varphi(t^*) = 1 + \epsilon t^{*2}$, $\epsilon > 0$]. On the other hand, the magnitude of the velocity overshoot is small in the y component of velocity S. For large blowing rates ($A = -30$) and for time t^*, the total enthalpy profile g first decreases near the wall to nearly zero and then increases to one as η increases. This implies that the fluid near the wall becomes hotter than the wall and that the heat is then transferred from the fluid to the wall and not from the wall to the fluid. This is caused by the Mach number/yaw/angle parameter λ. Also, the boundary-layer thickness increases rapidly as the blowing rate increases.

The effect of parameter λ on the velocity and total enthalpy profiles f', S, and g is presented in Fig. 2. It is seen that increase in λ causes a substantial increase in the magnitude of the velocity overshoot in f' and S. Also, the total enthaply profile g is significantly affected by λ. It is also observed that in the presence of large blowing, the effect of ω on the velocity and total enthalpy profiles is small.

The effect of blowing ($A<0$) on the skin-friction and heat-tranfer parameters f''_w, S'_w, and g'_w has also been studied, but not shown here for the sake of brevity. As expected, the skin friction and heat transfer reduce as the blowing rate ($A<0$) increases. However, the heat-transfer parameter g'_w tends to zero faster than the skin-friction parameters f''_w and S'_w as the blowing rate increases. However, the skin-friction and heat-transfer parameters increase with time t^* as the freestream velocity distribution accelerates [$\varphi(t^*) = 1 + \epsilon t^{*2}$, $\epsilon > 0$]. If we consider the decelerating flow [$\varphi(t^*) = 1 - \epsilon t^{*2}$, $\epsilon > 0$], then f''_w, S'_w, and g'_w decrease as time t^* increases.

The effect of blowing ($A<0$) on the skin-friction and heat-transfer parameters (f''_w, S'_w, g'_w) for $\varphi(t^*) = [1 + \epsilon_1 \cos(\omega^* t^*)] \div (1+\epsilon_1)$ has been studied. It is observed that the skin-friction parameters f''_w and S'_w respond more to the fluctuations in the freestream velocit than the heat-transfer parameter g'_w.

Conclusions

Blowing strongly affects the skin friction, heat transfer, velocity, and total enthalpy profiles. There is an overshoot in the x component of the velocity (chordwise velocity) and the magnitude of the velocity overshoot increases as the blowing rate or Mach number/yaw/angle parameter λ increases. Velocity overshoot is also observed in the y component of the velocity (spanwise velocity). Also, the boundary-layer thickness rapidly increases with the blowing rate. The heat transfer tends to zero more rapidly as compared to skin-friction parameters as the blowing rate increases.

References

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